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GARCH Analyses of Risk and Uncertainty in the Theories of the Interest Rate of Keynes and Kalecki

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Abstract

This study attempts to identify uncertainty in the long-term rate of interest based on the controversial interest rate theories of Keynes and Kalecki. While Keynes stated that the future of the rate of interest is uncertain because it is numerically incalculable, Kalecki was convinced that it could be predicted. The theories are empirically tested using a reduced-form GARCH-in-mean model assigned to six globally leading financial markets. The obtained results support Keynes's theory – the long-term rate of interest is a nonergodic financial phenomenon. Analyses of the relation between the interest rate and macroeconomic variables without interest uncertainty are thus seriously incomplete.

Keywords: uncertainty, interest rate, Keynes, Kalecki, GARCH

JEL classification: B26, C58, E43, E47

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1. Introduction

Expectations formed under conditions of uncertainty play a key role in Keynes's *General Theory of Employment, Interest and Money* (Keynes, 1936), particularly in his theory of the interest rate (Wheatherson, 2002). By contrast, Kalecki widely neglected uncertainty in his work, including in his theory of the rate of interest. Keynes's concept of numerically incalculable uncertainty is akin to Knight's 'true' uncertainty in distinction to calculable 'risk' (Knight, 1921; Sakai, 2016). According to Kalecki, the expected rate of interest appears as a probabilistic function of present and past observations (Kalecki, 1936, 1939, 1954). Barkley Rosser Jr. (2001) noted that Keynes's concept of uncertainty was turned into a concept of objectively measurable risk by Tobin. Indeed, Kalecki's view on risk is close to Tobin's view, which prevails in contemporary macroeconomics and in the growing literature on the intersection between uncertainty and financial markets since the 1980s (for an overview, see Hartzmark, 2016).

When true uncertainty cannot be measured, any attempt to do so – be it in research or in business practice – appears as an oxymoron. This may explain why in the literature the works of Keynes and Kalecki are compared mostly on theoretical grounds. However, the purpose of my study is to enrich this debate with an exploration of likely undiscovered regions, namely, the empirical examination of the two contradictory theories of Keynes and Kalecki. The question is whether the future long-term rate of interest is determined by objective facts from the present and past. If this is the case, Kalecki's views might hold and one can thus reliably calculate the risk of investing money in real or financial assets. If not, the future of the interest rate is uncertain in the Keynesian-Knightian sense. The empirical analysis in this paper is also new in that it applies a reduced-form non-linear generalised autoregressive conditional heteroscedasticity (GARCH) model (Bollerslev et al., 1994) and ignores the specific restrictions implied by various other models. GARCH models have been applied to predict the risk of a financial investment (Hartzmark, 2016) and to forecast policy rates of the central bank under the condition of uncertainty (Chuderewicz, 2002; Lanne and Saikkonen, 2003; Mandler, 2007).

The examination in this study takes the following form. The next section provides a brief characterisation of theories of the rate of interest according to Keynes and Kalecki, with an emphasis on the empirical application used in the following sections. Section three introduces the GARCH framework. Section four describes data on the long- and short-term interest rates of six economies with relevance to global financial markets. Section five presents and discusses the empirical findings. Conclusions are provided in Section six. The analysis finds evidence for 'true' uncertainty in the long-term rate of interest and underlines how topical Keynes's reasoning still is.

2. Psychology vs objective facts

Keynes introduced the rate of interest in Chapter 11 of the *General Theory* as a factor that, together with the marginal efficiency of capital, induces investment. At first glance, his statements appear to be the basis for standard investment theory, where investment is falling in the interest rate. However, in the following chapters, he made clear that he has another aspect in mind. He was sceptical that the probability of the future rate of interest can be mathematically derived from present interest rates: 'Just as we found that the marginal efficiency of capital is fixed, not by the 'best' opinion, but by the market valuation as determined by mass psychology, so also expectations as to the future of the rate of interest as fixed by mass psychology have their reactions on liquidity-preference – but with this addition that the individual, who believes that future rates of interest will be above the rates assumed by the market, has a reason for keeping actual liquid cash, whilst the individual who differs from the market in the other direction will have a motive for borrowing money for short periods in order to purchase debts of longer term', (Keynes 1936: p.108).[1] Practically, the existence of 'subjective factors' such as liquidity preference and mass psychology has the consequence of not only the rate of interest mattering for investment activities but also the state of uncertainty about the future rate of interest, in so far as the standard investment function is incomplete and misleading. Investment decisions and activities depend on the state of uncertainty and may even decline when the rate of interest is low.

The sense in which Keynes was using the term 'uncertain' is that '…in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth-owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know' (Keynes, 1937: p.214). 'Nevertheless,' he continued, there is the 'necessity for action and for decision' (ibid.), which is guided by mass psychology and animal spirits. He uses the term 'uncertainty' according to his book *A Treatise on Probability* (Keynes, 1921), where he defined cases of 'well-ordered' series beyond the two poles of 0% probability (= impossibility) and 100% probability (= certainty), where non-numerical probabilities may exist or where probabilities are not comparable.[2] This definition comes close to what F. H. Knight defined in his book *Risk, Uncertainty and Profit* of the same year (Knight, 1921) as 'true' uncertainty distinct from 'risk.' Risk is numerically calculable, as with Keynes's line between impossibility and certainty (see also Sakai, 2016). Cases of true uncertainty are not reduced to rare 'black swan' events such as sudden financial crises like that of 2007-2008, but can be found in 'well-ordered' and well-kept time series. In statistical language, such time series are nonstationary and have a unit root, meaning that after a shock, the series does not return to its pre-shock level. Paul Davidson referred to this phenomenon as nonergodicity (Davidson, 1982), which is typical for complex social systems, wherein the economy is a subsystem with close connections to other subsystems. If the rate of interest reflects uncertainty in a complex social system, its volatility, as I will argue later, affects the rate itself. Minsky, who was familiar with the financial sector, similarly judged that '…uncertainty implies that views about the future can undergo marked changes in short periods of time' (Minsky, 1977

[1] The term 'debt' refers to long-term securities such as bonds.

[2] This applies in the sense that one event is more probable than another event.

[1984: p.62]). Hence, entrepreneurs might change their decisions regarding holding liquidity or investing in financial and real assets often, even on a daily basis, based on changes in their liquidity preferences.

Kalecki presented the alternative hypothesis. His intention was to discover ‘objective’ laws governing the cyclicity of the capitalist economy. He assumed that these laws can be discovered from the hard facts of past experiences and recorded observations if one reduces the complexity of the system. His suspicion that a social system such as the capitalist system is governed by objective laws seems to have been an echo of 19th-century beliefs held across the European continent that economics has the character of a kind of social physics – with a specific sympathy for the Marxist school of thought (Lopez-Mott 1999: p. 293;Toporowski, 2013: pp. 43-55). Thus, his business cycle theory can be characterised as deterministic and as neglecting uncertainty.

Kalecki’s critique of Keynes’s theory of interest was derived from investment. In his *Theory of Economic Dynamics,* Kalecki focuses on investment decisions made prior to investment activities (Kalecki, 1954: pp. 96-98), which are hence an exogenously given determinant for output and employment. While Keynes held that frequent changes in the rate of interest provoke frequent corrections in investment activities, Kalecki rejected this argument. Instead, he maintained that investment expenditure automatically generates the same amount of savings (= gross profits) in the same period necessary for its financing: ‘Thus, investment 'finances itself' whatever the level of the rate of interest is’ (Kalecki, 1954: p.73). He concludes that entrepreneurs do not often change their investment plans, particularly not when they assess occurrences as of short-term duration (Lopez and Mott, 1999: p. 295).

In Chapter 9 of the *Theory of Economic Dynamics,* Kalecki stated that investment decisions are driven by the self-financing strength of the firm (its savings), the expected increase in profits while new installed equipment is in use, and the expansion of productive capacities. Unlike for Keynes, the rate of interest is not considered a factor for investment decisions; instead, the profit rate plays the key role. The cyclicity results from deviations between expected and actual profits, and these deviations stem from conflicts between capitalists and workers over the distribution of profits. This does not mean that the rate of interest is not relevant for investment at all. The rate of interest decides how profits are distributed between entrepreneurs and rentiers and thus determine indirectly investment (Osiatyński, 2019: p. 307). The long-term rate also plays a role in the long-term growth of the capitalist economy but not in its cyclicality or other short-term fluctuations. The latter issue was one of those openly debated between Keynes and Kalecki.[3]

The following quotation (Kalecki, 1954: p. 54) illustrates the relevant difference between Kalecki and Keynes on the rate of interest: ‘We shall argue that … the long-term rate is determined by anticipations of the short-term rate based on past experience and by estimates of the risk involved in the possible depreciation of long-term assets.’ Kalecki did not distinguish between the various motives for holding liquidity and thus did not consider a specific role for liquidity preference and uncertainty. The expected *average* short-term rate of interest plus a risk factor determines the long-term rate of interest. While in his *Treatise on Money*, Keynes (1930: p. 315, pp. 357-358) also assumed that the short-term rate ‘drives’ the long-term rate (Akram, 2020), his considerations in the later *General Theory* do not exclude a reverse causality: The long-term rate is governed by sentiments on capital and financial markets under conditions of uncertainty and mass psychology, which can be transmitted to the short-term rate through

[3] For a review of arguments between Keynes and Kalecki, see Lopez and Mott (1999, pp. 295-296).

the liquidity preferences of money holders.[4] Thus, Kalecki believed that entrepreneurs and any other market participants are able to predict the future rate of interest with calculable probability on grounds of present and past observations. What for Keynes was uncertainty was for Kalecki risk.

In Chapter 7 of his *Theory of Economic Dynamics*, Kalecki presented his view of how risk assessment proceeds: a market participant, faced with the decision to hold his liquidity in short-term money-market paper or to invest in a long-term bond, has to compare the long-term rate with the expected short-term rate. The market participant can reliably assume that a hike of the short-term rate in one period will be corrected in the following period. Kalecki formally showed that the expected long-term rate is stable compared to the expected short-term rate – a claim he had already raised in much earlier work (1936 (1936 [1990],1939 [1990]). The brake is a risk coefficient defined as the numerically fixed probability of a fall in the market value of the bond to a minimum level in the relevant period, or conversely, an increase in its rate of return to a maximum.

4 Compared to the *General Theory* of 1936, the earlier work *A Treatise on Money*, in particular Vol. II, is more monetary policy-oriented.

3. Methodological issues

Kalecki added a statistical illustration to his theory of the long-term rate of interest. He estimated a linear regression with least squares of the following form:

$$\mathrm{r} = \frac{1}{\bar{g}}\rho_t^e + \bar{g} + \mu_t \tag{1}$$

where *r* is the yield – the nominal rate of interest – of long-term security, in his case a British government bond ('consol'), and μ is the identically distributed stochastic error term.[5] The long-term rate is a function of the *expected* rate of interest ρ^e on short-term papers and a risk coefficient $\bar{g}$. The risk coefficient is constant in the period under consideration and consists of two elements: (i) the minimum value of the bond or maximum of its yield assumed by the investor in the period of holding alternatively short-term papers being larger than one, and (ii) the inconvenience and – negligible – costs of holding a short-term paper compared to a long-term paper (Kalecki, 1954:p. 81). While the former element is positive by nature, the latter one is negative. Thus, $g > 1$, and $1/g < 1$ is the 'brake' mentioned above. The investor assumes the *actual* short-term rate to be mean reverting over the considered period (stationarity assumption). Then, the risk of the bond value falling below the expected minimum is equal to the standard error of the regression, and the expected error term exhibits the risk in the dependent variable. In technical terms, the errors of such a backward-looking regression need to have a zero mean and a constant variance σ^2 (homoscedasticity); otherwise, the regression coefficients will be distorted, and the *t*-statistics will not be usable. Of course, one must ask whether such a complexity-reduced system exists in the real world of finance. Many financial series are not homo- but heteroscedastic with the expected size of the error, depending on its past values, and with a volatile standard deviation and variance. Then, there will be no reliable forecasts.

This study applies a GARCH-in-mean (MGARCH) model where the expected variance is conditional to new information, and which is more appropriate to complex economic systems. The MGARCH model was selected from the pool of numerous GARCH models, because it consists of a conditional mean equation and a conditional variance equation. The conditional mean equation for the determination of the long-term rate has the following form:

$$r_t = C_0 + C_1\rho_t^e + C_2\sigma_t^2 + \varepsilon_t \tag{2}$$

where the σ^e is the exogenous short-term interest rate. The Kalecki equation (1) is augmented by the endogenous variable σ^2 – the conditional variance – expected in period *t*. This GARCH term is estimated from the error terms in equation (2) according to:

$$\sigma_t^2 = h_0 + h_1\varepsilon_{t-1}^2 + \cdots + h_p\varepsilon_{t-p}^2 + g_1\sigma_{t-1}^2 + \cdots + g_q\sigma_{t-q}^2 \tag{3}$$

5 This is not included in Kalecki's equation.

The conditional variance is the moving average of the squared past error terms and the lagged squared values of itself. It is conditional on 'new' information about the deviation of the error from its expected value in the previous periods. Thus, it defines the error (risk) expected for the present period. A higher variance depicts a riskier period. 'Riskier' periods are not scattered randomly across the examined time series but appear as volatility clustering, which may be explained as the effect of mass psychology among market participants. A positive value indicates that risk aversion prevails among market participants and that they require a risk premium. A negative value exhibits a prevalence of risk-friendly investors who accept a risk discount on the bond yield because they expect a higher future market value of the bond. Equation (3) reveals that the conditional variance in a MGARCH model is not a stochastic, but a pre-determined term. This carries the danger of spurious regressions, when the model is run with high-frequency data (daily and shorter. A Markov-switching GARCH (MSGARCH) with a stochastic variance could be an alternative option. However, this is impossible with a GARCH-in-mean model because it decouples the mean and volatility estimation model (Ardia et al., 2019: p. 3), while this study seeks to explore the impact of exogenous determinants (here, the short-term rate) on the long-term rate in the mean equation. In addition, with the use of monthly data, the danger of spurious regressions is reduced.

Equation (3) describes the GARCH (p, q) model with *p* lagged ARCH and *q* lagged GARCH elements. The ARCH terms $\varepsilon_{t-1}^2 \dots \varepsilon_{t-p}^2$ are the squared lagged error terms taken from the mean equation. They represent the moving average of past error terms as adjustments of the actual long-term rate to a shock in the previous period. The GARCH terms $\sigma_{t-1}^2 \dots \sigma_{t-q}^2$, i.e., the lagged conditional variances from the previous period, measure the persistence in volatility carried from the preceding period. Because GARCH (1, 1) models prove to be the most robust models, they are applied here, provided that $h_1 > 0$ and $g_1 > 0$ and $\sum(h_i + g_{1)} < 1$. The sum of coefficients shows whether and how quickly a shock is persistent or dies over time (Chan, 2010). When $\sum(h_i + g_{1)} < 1$, the GARCH j process is weakly stationary and ergodic, and only then can the long-run average of the variance be used for risk assessment. If the sum is close to unity from below, the mean reversion process occurs very slowly. If the sum is equal to one, the conditional variance is not mean reverting. The same applies when it is larger than one, denoting an 'explosive' asymptotic dynamic – the long-term rate infinitely increases or falls. In both circumstances, a risk assessment is not possible; we then enter the realm of Keynesian or true uncertainty and nonergodicity. This is mostly the case when the variance has a unit root (Diebold, 1986). When the unit root is integrated in the GARCH model (IGARCH), $\sum h_{ij} + g_{1j}$ is restricted to 1, which improves the efficiency of the estimates. However, a previous change in the dependent variable persists, and the variable does not return to its previous level.

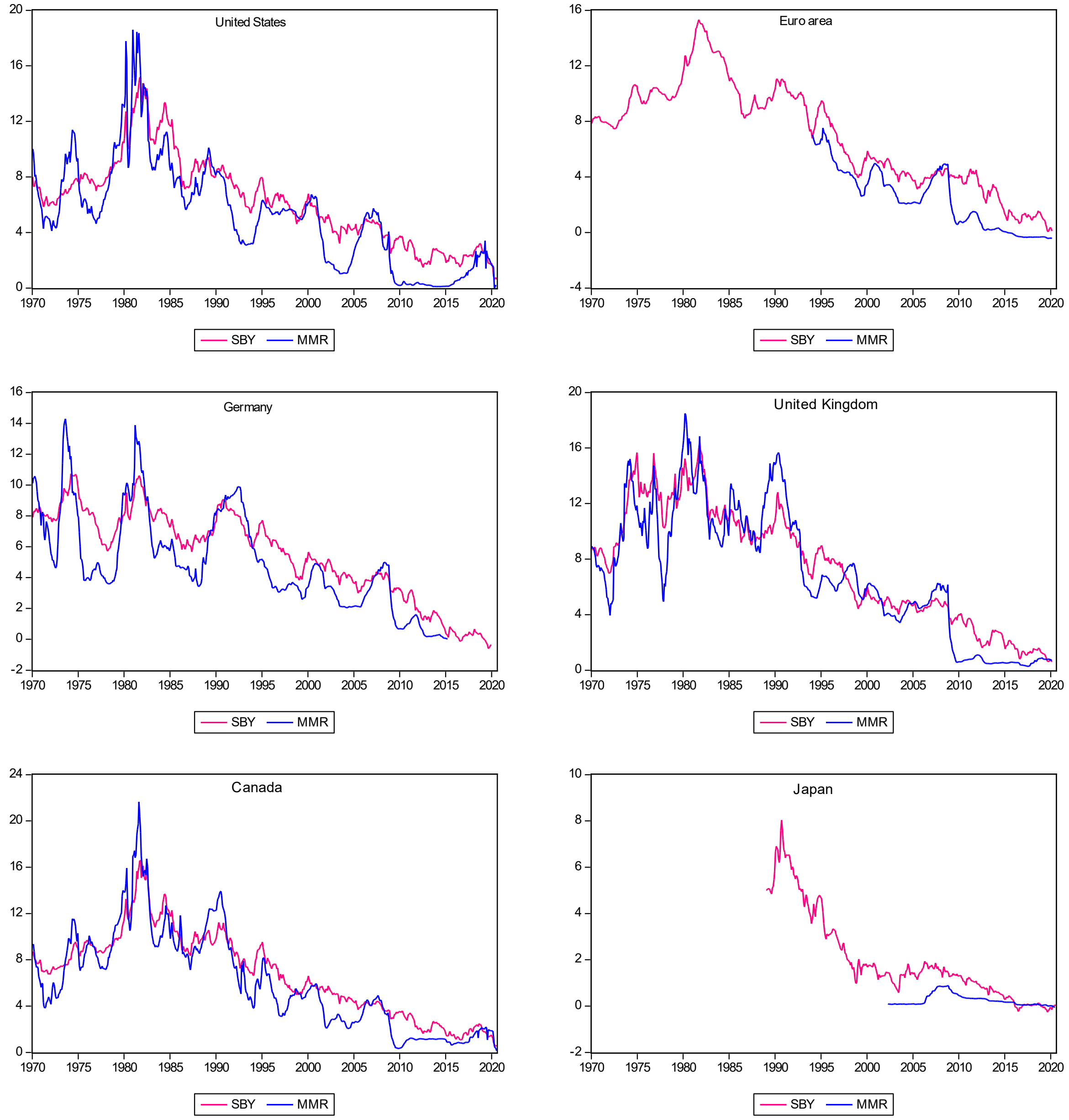
4. Data and their properties

The long-term rate of interest is proxied by the monthly nominal yields of ten-year sovereign bonds (SBY), and the short-term rate is proxied by monthly data of the nominal three-month interbank offered rates (MMR) of six financial markets: the United States, the euro area, Germany, the United Kingdom, Canada and Japan. These entities were selected according to the global role of their currencies as reserve currencies held in significant quantities by other central banks and large international companies. All data are taken from the Federal Reserve Bank of St. Louis (Federal Reserve Economic Data – FRED; https://fred.stlouisfed.org), in percentages. The original data were seasonally adjusted with Census X12 or X13. The sample periods run from 1970:1 to 2020:7, with a maximum of 607 observations. The expected short-term rates (MMREs) are calculated as the moving average of the past 12 months, assuming that market participants wish to hold a long-term paper for at least one year.

A review of the monthly patterns of rates of interest reveals convergence between financial markets (Figure 1) triggered by arbitrage in the globalised financial world. The normal yield curve holds: short-term rates tend to be below long-term rates, except in Japan from 2016:1. The inverted yield curve for Japan at the sample end signals that investors expect a recession and the market value of short-term papers to plummet soon.

The figure also shows a decrease in the interest rate in some periods followed by another decrease, and in other periods, an increase followed by another increase. Seemingly, individual investment behaviour in bond markets is influenced by mass psychology or herding, leading to serial correlations, ARCH elements and unit roots in the observed series, all of which underline the appropriateness of a GARCH model. This conclusion is confirmed by an ARCH test of the residuals of first-order autoregressive models of the long- and short-term rates. The *p*-values (Table 1) reject the null of no ARCH at the 1% or 5%level for Germany at the 10% level (with two lags also at the 1% level).

Figure 1 / Monthly pattern of ten-year sovereign bond yields (SBY) and three-month money-market rates (MMR); seasonally adjusted



Source: Author's drawing, with EViews and data from the Federal Reserve Bank of St. Louis.

Table 1 / ARCH tests of residuals of first-order autoregressive estimations; long-term government bond yields (SBY) 3 and three-month money-market rates (MMR) – Obs.*R-squared residuals.[a]

	United States	Euro area	Germany	United Kingdom	Canada	Japan
SBY	34.613***	32.168***	2.932	81.373***	43.657***	52.651***
MMR	11.724***	48.773***	55.168***	7.183***	23.583***	4.520**

Significance levels: *** 1 %, ** 5 %. [a] Tests with one lag.
Sources*:* Author's calculations, with EViews and data from the Federal Reserve Bank of St. Louis.

Standard tests detect unit roots in the levels of the long- and short-term rates of interest (Table 2); first differences are stationary (not shown in table). Cointegration would be an option (Akram and Das, 2017) with ARDL, but seems not adequate in dealing with risk and uncertainty. The same conclusion relates to regressions with first differences, which are not appropriate to detect uncertainty. Thus, the use of level data is standard in risk assessment and uncertainty models.

Table 2 / Unit root tests for SBY levels, p-values

	United States	Euro area	Germany	United Kingdom	Canada	Japan
ADF	0.184	0.452	0.03	0.553	0.422	0.714
Phillips-Perron	0.275	0.418	0.151	0.492	0.475	0.714
Im, Pesaran, Shin	0.275	0.703	0.418	0.491	0.422	0.559

[a] Individual intercept, no trend. Legend: SBY: ten-year government bond yields.
Sources*:* Author's calculations, with EViews and data from the Federal Reserve Bank of St. Louis.

The maximum values of the SBY series are high compared to the minimum values, exhibiting the presence of spikes in the series (Table 3). Jarque-Bera (JB) statistics show strong and significant non-normal distributions, serving as another justification for GARCH application. Additionally, JB statistics suggest considering the inclusion of an error distribution parameter. However, kurtosis is not too far from a normal distribution (3), except for the United Kingdom. From the standard deviation, MMR volatility is greater than SBY volatility in four of the six markets, but lower for the euro area and Japan, which goes against Kalecki's predictions.

Table 3 / Descriptive statistics: ten-year government bond yields (SBY), %, monthly

(in brackets: standard deviation of MMR)

	United States	Euro area	Germany	United Kingdom	Canada	Japan
Maximum	15.167	15.326	10.730	16.472	16.583	8.042
Minimum	0.626	0.055	-0.592	0.617	0.529	-0.253
Std. dev.	3.083	3.689	2.856	4.082	3.575	3.083
(MMR)	*(3.802)*	*(2.184)*	*(3.111)*	*(4.620)*	*(4.267)*	*(0.244)*
Skewness	0.509	0.064	-0.408	0.106	0.251	0.509
Kurtosis	2.891	2.089	2.216	1.891	2.420	2.891
Jarque-Bera	26.486***	21.225***	32.023***	31.968***	14.892***	26.486***
No. of obs.	607	602	600	602	607	379

Significance levels: *** 1 %, ** 5 %.
Sources: Author's calculations, with EViews and data from the Federal Reserve Bank of St. Louis.

5. Results and discussion

For each financial market, two GARCH-in-mean (1, 1) models are estimated, for which the mean equations are:

$$SBY_t^i = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + z_t \quad (4.1)$$

$$SBY_t^i = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 MMRE_t^i + \pi_t \quad (4.2)$$

Model 4.1 estimates the pure effect of sentiments on the bond markets on the long-term rate under the – unrealistic – assumption that there are only bonds as alternative to holding liquidity. The constant term includes all time-invariant effects of the monetary order and the liquidity premium on the long-term rate. The latter is the convenience of holding cash, which Keynes defined as the own-rate of interest of money and serves as the minimum rate of interest (Keynes, 1936: Ch.17). In regressions, the term should show a positive sign. The calculation of the conditional variance σ^2 follows the pattern of equation (3), and z_t is the error term. A negative or positive sign of α_1 follows the explanations above. When the error is normally distributed and homoscedastic and the sum of the lagged ARCH and GARCH terms is below one, a reliable risk assessment is possible. Model 4.2 represents the Keynesian-Kaleckian theory that the short-term rate of interest determines the long-term rate with a certain bias towards Kalecki's *expected* short-term rate. In this model, the sign of the constant β_0 is not predetermined. It represents the combined effects of the liquidity premium (positive) and of Kalecki's assumption of the inconvenience of holding a short-term paper compared to a long-term paper (negative). If the constant term is negative, Kalecki's assumption would hold. The variance σ^2 measures the volatility of the spread between the yields of the two asset classes considered here. The coefficient β_2 is expected to be positive and less than one, and π_t is the error term. It is a reduced-form model because β_2 incorporates expectations on inflation and the output gap and the reaction of the central bank.

Table 4 / Model 4.1 – GARCH (1,1) estimation results

Dep. Var.: SBY	United States	Euro area	Germany	United Kingdom	Canada	Japan
Log (GARCH)	0.384***	0.080***	-0.039	-0.027***	-0.004***	-0.002***
Constant	5.509***	8.391***	6.032***	7.648***	7.456***	1.322***
St. E. of regression	3.067	4.075	2.862	4.086	3.642	1.067
Unit root in variance?[a]	3 of 3	3 of 3	3 of 3	2 of 3	3 of 3	2 of 3
Sum ARCH & GARCH	1.070	1.059	0.973[b]	1.083[b]	1.020	1.045
Jarque-Bera stat.	75.685***	62.608***	75.058***	82.894***	76.690***	25.874***
Kurtosis	3.155	1.460	1.276	1.185	1.301	1.640
ARCH LM test[c]	74.396***	17.581***	4.124**	0.089	19.599***	1.932
Sample	1970:1 – 2020:7	1970:1 2020:2	1975:12 – 2019:12	1970:1 – 2020:2	1970:2 – 2020:7	1994:2 – 2020:7
No. of obs.[d]	607	602	529	602	606	318

Significance levels: *** 1 %, ** 5 %. [a] Im, Pesaran, and Shin; Fisher-ADF; Fisher-PP; p-values ≤ 0.05. [b] GARCH-term negative. [c] Obs.*R-squared res. (lag1). [d] After adjustments.
Sources: Author's calculations, with EViews and data from the Federal Reserve Bank of St. Louis.

Tables 4 to 7 present the estimation results. Only *p*-values of ≤ 0.05 are indicated. The constant term is positive and significant in all models, suggesting the prevalence of a liquidity premium. Differences between the countries are probably due to their monetary order and other institutional habits and conventions.

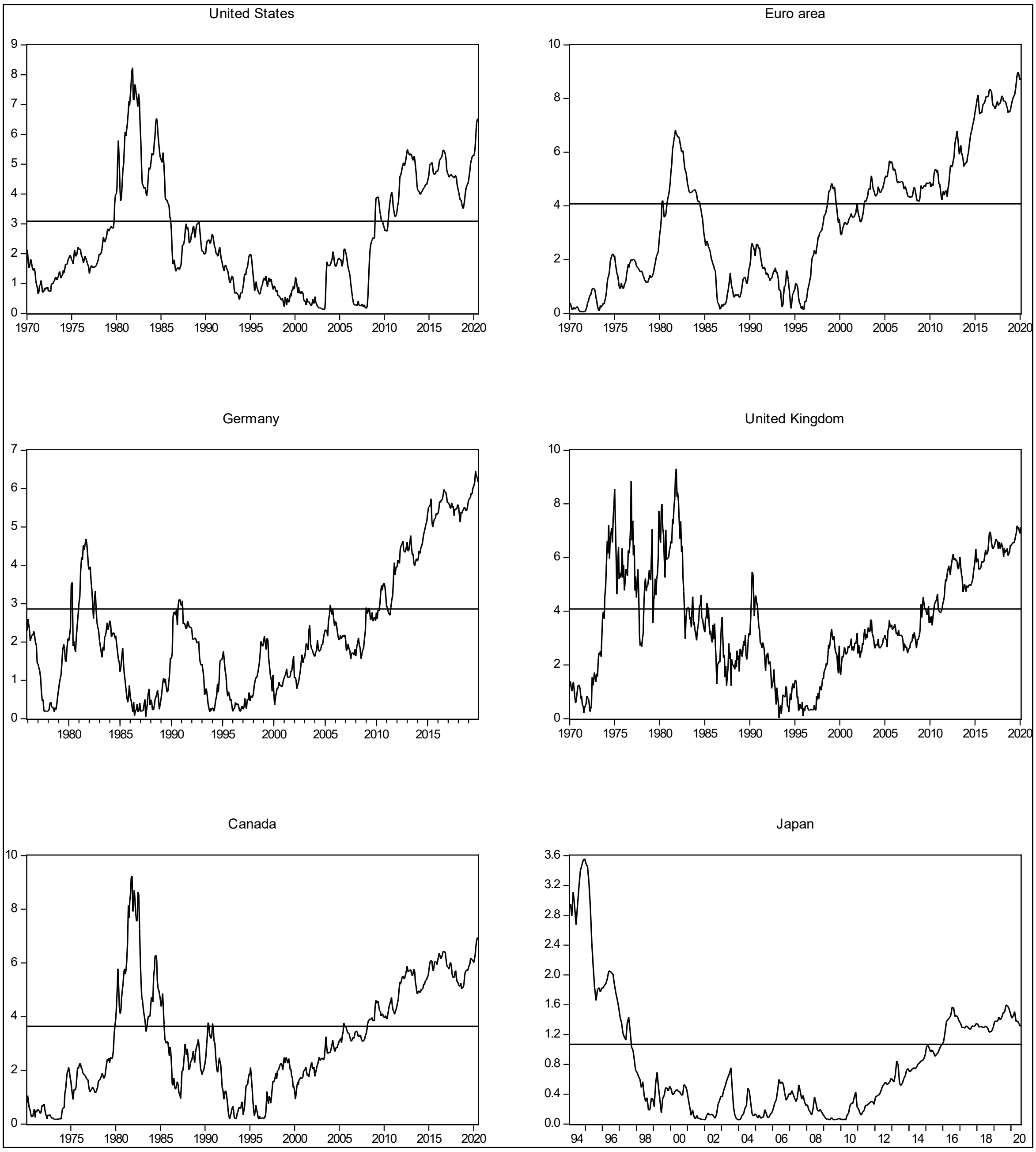
The estimation results for Model 4.1 are reported in Table 4. The conditional variance is significant for all financial markets and shows the presence of liquidity preference. In four of the six markets, risk-friendly participants seem to dominate (negative sign). This is probably because participants expect higher future returns in a recessionary stage of the economy and accept a risk discount. A positive sign appears only for the United States and euro area financial markets; participants expect lower returns in the future and require a risk premium. The risk factor or conditional standard deviation – the squared root of the variance – of the long-term rate in period *t,* expected from innovations or news in period *t-1*, is highly volatile, as shown in Figure 2. This diverges from the Kalecki model in equation (1) with its constant standard deviation and expected error term. By contrast, the GARCH model reveals that the expected error term of the next period is uncertain. The standard error of regression of Model 4.1 – the horizontal straight line – is close to the standard deviation of the error term in the linear least squares model and reveals how misleading the assumption of a constant risk factor might be in investment decisions when this factor is changing at every point in time. Figure 2 also shows volatility clustering, reflecting the impact of mass psychology on the long-term rate of interest. The conditional variance process is clearly nonstationary, as indicated by the results of the three standard unit root tests applied. The existence of a unit root in the conditional variance explains a sum of the ARCH and GARCH terms of larger than one, indicating an infinitely increasing or decreasing variance and hence a long-term rate. A forecast is impossible, and the parameters of the estimation are probably distorted. The exception is Germany, with a sum of terms of close to one from below; however, the GARCH term g_1 violates the non-negativity condition, as is the case for the United Kingdom. The rejection of a normal distribution of the residuals by the Jarque-Bera test and of homoscedasticity by the ARCH LM test in four countries, with the United Kingdom and Japan being exceptions, introduces the possibility of bias in the estimated means. Hence, predicting the long-term rate becomes uncertain in the Keynesian-Knightian sense.

Table 5 represents the IGARCH version of Model 4.1. The model imports the unit root into the conditional variance and restricts $\sum(h_i + g_{1)} = 1$. The variance process is no longer infinite, as innovations in period *t-1* are persistently transmitted to all following periods but remain nonergodic. The restriction has the advantage of generating more reliable model parameters. However, robust results can be obtained merely with a substantial reduction in the sample period. Otherwise, h_1 and g_1 are extremely high in absolute terms, and the GARCH term g_1 becomes negative. The most conspicuous result of the reduced sample is that the conditional variance is now positive in all markets, in contrast to Table 4. The only exception is found from the United States financial market with an insignificant variance, which also causes the unit root to disappear.[6] The model restriction cannot eliminate the unit root in the other five cases with a significant variance. Additionally, the non-negativity condition for the (1, 1) model is fulfilled for Germany and the United Kingdom. The forecast quality – assessed in terms of the JB statistic and ARCH LM test – remains low in all markets, although the ARCH elements could be removed from the residuals for the United Kingdom and Japan. The residual distribution is mostly leptokurtic (except in the United States market), exhibiting an elevated vulnerability of market participants' responses to unexpected new information. Leptokurtosis is coupled with a negatively

6 A further reduction of the sample (not shown here) shows positive and significant variance with a unit root.

skewed distribution, meaning that a shock provokes a stronger positive than negative response of the long-term rate.

Figure 2 / Conditional standard deviation (Model 4.1)



Sources: Author's drawing, with EViews and data from the Federal Reserve Bank of St. Louis.

Table 5 / Model 4.1 IGARCH (1,1) estimation results

Dep. Var.: SBY	United States	Euro area	Germany	United Kingdom	Canada	Japan
Log (GARCH)	-0.000	0.352***	0.708***	0.223***	0.153***	0.093***
Constant	2.357***	2.355***	2.968***	2.496***	2.191***	1.392***
St. E. of regression	0.881	0.762	0.335	1.226	0.611	1.050
Unit root in variance?[a]	0 of 3	3 of 3	1 of 3	3 of 3	1 of 3	2 of 3
Jarque-Bera stat.	5.437	79.366***	290.769***	7088***	6.517**	1747***
Kurtosis	2.360	7.482	10.150	34.617	3.072	14.111
ARCH LM Test[b]	19547***	12.040***	65.303***	0.120	15.517***	0.200
Sample	2007:5 – 2020:7	2013:8 – 2020:2	2011:3 – 2019:12	2007:8 – 2020:2	2010:2 – 2020:7	1994:2 – 2020:7
No. of obs.[c]	159	79	106	151	126	318

Significance levels: *** 1 %, ** 5 %. [a] Im, Pesaran, and Shin; Fisher-ADF; Fisher-PP; p-values ≤ 0.05. [b] Obs.*R-squared res. (lag1). [d] After adjustments.
Sources: Author's calculations, with EViews and data from the Federal Reserve Bank of St. Louis.

Table 6 presents the results of model 4.2 with the *expected average* short-term rate of interest used as an additional explanatory variable. Now, the conditional variance appears highly significant and positive in all markets. As predicted by theory, the short-term rate has a positive sign and is significantly different from zero. However, there is merely a loose relationship between the short- and long-term rates. For Japan, a 1% increase in the short-term rate is related to a more than 1% increase in the long-term rate. This result opens the perspective of a reversed causality – from the long-term to the short-term rate – in a recessive or stagnating economy, such as Japan. Despite the absence of a unit root in the variance in most cases, the sum of the ARCH and GARCH terms is still larger than one, except for the United States; however, for the latter, the non-negativity condition is violated. In sum, there is more evidence for true than for measurable uncertainty, and the forecast quality of the models remains poor with respect to the non-normal distribution of all residuals and serial correlation in four of the six cases.

Table 6 / Model 4.2 GARCH (1,1) estimation results

Dep. Var.: SBY	United States	Euro area	Germany	United Kingdom	Canada	Japan
Log (GARCH)	0.050***	0.308***	0.221***	0.128***	0.286***	0.191***
MMRE	0.818***	0.819***	0.735***	0.656***	0.577***	2.113***
Constant	1.862***	2.303***	0.501***	0.162***	1.478***	0.609***
St. E. of regression	1.165	0.827	2.074	2.484	2.432	0.235
Unit root in variance?[a]	0 of 3	1 of 3	0 of 3	0 of 3	0 of 3	0 of 3
Sum ARCH & GARCH	0.962[b]	1.048	1.003	1.120	1.133	1.091
Jarque-Bera stat.	20.312***	100.593***	53.437***	24.491***	145.132***	103.145***
Kurtosis	2.872	4.447	2.056	2.236	4.435	6.805
ARCH LM Test[c]	1.847	20.395***	13.517***	12.773***	33.012***	1.684
Sample	1970:1 – 2020:7	1994:1 – 2020:7	1978:12 – 2019:12	1980:12 – 2020:2	1981:8 – 2020:7	2006:4 – 2020:6
No. of obs.[d]	596	314	493	471	468	171

Significance levels: *** 1 %, ** 5 %. Legend: MMRE: expected 3months money market rate. - [a] Im, Pesaran, and Shin; Fisher-ADF; Fisher-PP; p-values ≤ 0.05. [b] GARCH-term negative. [c] Obs.*R-squared res. (lag1). [d] After adjustments.
Sources: Author's calculations, with EViews and data from the Federal Reserve Bank of St. Louis.

Table 7 presents the results of Model 4.2 with the IGARCH restriction. Again, we observe a change in the sign for the conditional variance, now from positive to negative. The unit root in Table 6 moves from the euro area to Germany. The model results confirm two findings of the former model: a loose relationship between short- and long-term rates and the possibility of a less stable long-term rate compared to the short-term rate (again in the case of Japan). The residual tests confirm the limited forecast quality of most models.

All models were additionally estimated for the expanded sample periods with a Student's *t* error distribution and a general error distribution (GED) parameter. The results do not add further nuance to the previous findings and are thus not reported here. Either the sum of ARCH and GARCH terms was found to be larger than one or the non-negativity conditions were violated; in some cases, a mixture of both emerged.

Table 7 / Model 4.2 IGARCH (1,1) estimation results

Dep. Var.: SBY	United States	Euro area	Germany	United Kingdom	Canada	Japan
Log (GARCH)	-0.032***	0.086***	0.452***	-0.069***	-0.074***	0.099***
MMRE	0.815***	0.940***	0.046***	0.392***	0.463***	1.873***
Constant	1.898***	1.685***	1.977***	1.258***	1.330***	0.364***
St. E. of regression	1.165	0.864	0.502	1.191	1.204	0.324
Unit root in variance?[a]	0 of 3	0 of 3	3 of 3	0 of 3	0 of 3	0 0f 3
Jarque-Bera stat.	0.756	15.694***	294.243***	3310***	0.486	25.381***
Kurtosis	2.831	2.579	10.040	19.231	3.045	4.508
ARCH LM Test[b]	21.113***	51.280***	43.936***	0.009	3.932**	0.416
Sample	1970:1 – 2020:7	1994:1 – 2020:7	2011:2 – 2019:12	1996:5 – 2020:2	1995:4 – 2020:7	2006:4 – 2020:6
No. of obs.[c]	596	314	107	286	304	171

Significance levels: *** 1 %, ** 5 %. Legend: MMRE: expected three-month money-market rate. - [a] Im, Pesaran, and Shin; Fisher-ADF; Fisher-PP; p-values ≤ 0.05. [b] GARCH-term negative. [c] Obs.*R-squared res. (lag1). [d] After adjustments.
Sources: Author's calculations, with EViews and data from the Federal Reserve Bank of St. Louis.

6. Concluding notes

Our study comes to an end with a possibly still fragmented mapping of a formerly undiscovered region, namely, the empirical validity of Keynes and Kalecki's theories of the rate of interest. Our 'fact-finding' mission using the GARCH tool collected more evidence for Keynes's theory. It is true that the concept of liquidity preference and Keynes's theory of the interest rate are disputed among post-Keynesians, owing to identified or assumed inconsistencies in his theory of money (see Watanabe, 2008 for his retrospective overview). Without contributing to this debate, my empirical study reveals signs of liquidity preference in the variance processes of six financial markets of global relevance. The coefficients for the variance term in the mean equations are in the overwhelming majority of the examined cases statistically significant. This validates the initial hypothesis that there is a correlation between uncertainty and expected returns. Keynesian or true uncertainty occurs when the conditional variance – the proxy for uncertainty – is not mean reverting, and this was found in the empirical analysis. Then, the ergodicity assumption for financial markets can be rejected. Some GARCH results raise doubts surrounding the assumed causality direction from the expected short-term rate to the long-term rate and call for a deeper investigation using appropriate methods (vector autoregression and impulse-response functions).

As mentioned above, Kalecki's critique of Keynes's theory of the interest rate was related to investment. Thus, the results obtained might also shed light on the investment theories of the two authors. Although Kalecki rejected the idea of an effect of the long-term rate of interest on investment due to its relative stability, empirical results show strong variability in the long-term rate due to uncertainty. Thus, Keynes's claim that frequent changes in the expected rate of interest affect investment seems plausible. The standard 'Keynesian' investment function is seriously incomplete without a variable that captures uncertainty – conditional variance in our investigation.

However, the reader should recognise that empirical science cannot create unambiguity. The economy is a nonlinear complex system that is sensitive to changes in initial conditions, and there are limits to the accuracy of any model, including that applied here (Engle et al., 2008). Therefore, the obtained results must be merely interpreted as provisional results, although the application of the GARCH-in-mean model proved successful. Further 'mapping' studies are necessary.

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